Phase Structures of Compact Stars in the Modified Quark-Meson Coupling Model

Chang-Qun Ma and Chun-Yuan Gao*
School of Physics, Peking University, Beijing 100871, China

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Abstract
The K^- condensation and quark deconfinement phase transitions are investigated in the modified quark-meson coupling model. It is shown that K^- condensation is suppressed because of the quark deconfinement when $B^{1/4}$ <202.2MeV, where B is the bag constant for unpaired quark matter. With the equation of state (EOS) solved self-consistently, we discuss the properties of compact stars. We find that the EOS of pure hadron matter with condensed K^- phase should be ruled out by the redshift for star EXO0748-676, while EOS containing unpaired quark matter phase with $B^{1/4}$ being about 180MeV could be consistent with this observation and the best measured mass of star PSR 1913+16. We then probe into the change of the phase structures in possible compact stars with deconfinment phase as the central densities increase. But if the recent inferred massive star among Terzan 5 with M>1.68$M_\odot$ is confirmed, all the present EOSes with condensed phase and deconfined phase would be ruled out and therefore these exotic phases are unlikely to appear within neutron stars.

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*Electronic address: gaocy@pku.edu.cn

Neutron stars are some of the densest objects in the universe since their masses are of the order of 1.5 solar masses while their radii are only of ~ 12 km[1]. Therefore, the density in the inner core of a neutron star could be as large as several times nuclear saturation density($\cong 0.17$ fm^{-3}) and the appearance of new phases other than normal nuclear matter is possible. Kaplan and Nelson proposed the possibility of kaon condensation by a chiral theory[2]. While Bodmer[3] and Witten[4] suggested that the strange quark matter phase, which was discussed by Itoh in 1970[5], might provide the absolutely stable form of the dense matter. Following their work, many authors have devoted to studying the kaon condensation and/or quark matter phase in neutron stars[6]. In this letter, neutron star matter will be investigated with novel EOSes and the possibilities of appearances of exotic phases in neutron stars are going to be discussed.

Predictions by models with quark effects would be preferred to those by models with only hadron degrees of freedom because neutron star matter is extraordinarily dense. While at the moment QCD is not realized in investigating neutron stars because of its nonperterbative features, it is worthwhile studying the neutron star with effective quark models. In 1988, Guichon proposed a novel model[7], the quark-meson coupling (QMC) model, where the 'quark effect' was incorporated. The model and its modified versions give satisfactory description for saturation properties of nuclear matter[8] and reproduce the bulk properties of finite nuclei well[9]. Recently, Panda, Menezes and Providência discussed the kaon condensation[10] and deconfinement phenomena[11] in neutron star within the QMC model, where the bag constant was fixed at its free-space value and the strange quark was unaffected in the medium and set to its constant bare mass value. But the QMC model predicts much smaller scalar and vector potentials for the nucleon than that obtained in the well established quantum hadrodynamics model. Jin and Jennings modified the QMC model by introducing a density-dependent bag constant so that large scalar and vector potentials are obtained without affecting its abilities in other aspects[12]. It is imaginable that the s quark mass should also be modified at the supernuclear density in the core of neutron stars. So an additional pair of hidden strange meson fields ($\sigma^*,\ \phi$), which had been proved that they can account for the strongly attractive $\Lambda\Lambda$ interaction observed in hypernuclei that cannot be reproduced by ($\sigma,\ \omega,\ \rho$) mesons only[13], are included in the modified quark-meson coupling model (MQMC)[14]. ($\sigma^*,\ \phi$) couple only to the s quark in the MQMC model and only to the hyperons in the QHD model. The improved MQMC model has been used to study kaon production in hot and dense hypernuclear matter[15].

In the present work, we shall extend the MQMC to investigate both the K$^-$ condensation and quark deconfinement phase transitions in compact stars at zero temperature. All of the three most possible phases, i.e. the hadronic phase (HP) with strangeness-rich hyperons, the condensation for negative charged kaon and the quark matter phase are considered.

Both baryons and kaon meson are described by static spherical MIT bags. Quarks are taken as explicit degrees of freedom, and are coupled to the meson fields. The nonstrange (u and d) quarks in the baryons and kaons are coupled to the well known σ, ω and ρ meson fields while the strange quark in the baryons and kaons is coupled to σ^* and ϕ only, because the former three pieces are built out of u-, d-quarks or their anti-counterparts and the later two are composed of strange quarks. Let the mean fields be denoted by σ, σ^* for the scalar meson fields, and ω_0, ϕ_0 and ρ_{03} for expectation valus of the timelike and the isospin three-component of the vector and the vector-isovector meson fields. In the mean field approximation the dirac equation for a quark field of flavor $q \equiv (u,\ d,\ s)$ in the bag for the hadron species $i \equiv$ (p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, K$^-$) is then given by

$$\begin{aligned}&\left[\mathrm{i}\gamma\cdot\partial - m_q + \left(g_\sigma^q\sigma - g_\omega^q\omega_0\gamma^0 - g_\rho^q I_{3i}\rho_{03}\gamma^0\right)\right.\\&\left.+\left(g_{\sigma^*}^q\sigma^* - g_\phi^q\phi_0\gamma^0\right)\right]\psi_{qi}(\vec{r},t) = 0.\end{aligned} \quad (1)$$

The normalized ground state is solved as

$$\psi_{qi}(\vec{r},t) = \mathcal{N}_q \exp\left(-\mathrm{i}\epsilon_{qi}t/R_i\right)\begin{pmatrix} j_0\left(x_{qi}r/R_i\right) \\ \mathrm{i}\beta_{qi}\vec{\sigma}\cdot\hat{r}j_1\left(x_{qi}r/R_i\right)\end{pmatrix}\frac{\chi_q}{\sqrt{4\pi}}, \quad (2)$$

where

$$\epsilon_{qi\pm} = \Omega_{qi} \pm R_i \left(g_\omega^q \omega_0 + g_\rho^q I_{3i} \rho_{03} + g_\phi^q \phi_0 \right), \quad (3)$$

$$\beta_{qi} = \sqrt{\frac{\Omega_{qi} - R_i m_q^*}{\Omega_{qi} + R_i m_q^*}}, \quad (4)$$

$$\Omega_{qi} = \sqrt{x_{qi}^2 + \left(R_i m_q^*\right)^2}, \quad (5)$$

with

$$m_q^* = m_q - g_\sigma^q \sigma - g_{\sigma^*}^q \sigma^*, \quad (6)$$

the effective mass of quark with flavor q; R_i is the bag radius of hadron species i; I_{3i} is the isospin projection for the hadron species i; x_{qi} is the dimensionless quark momentum and it can be determined from the boundary condition on the bag surface by the eigenvalue equation

$$j_0(x_{qi}) = \beta_{qi} j_1(x_{qi}). \quad (7)$$

In Eq. (3), + sign is for quarks and − sign is for antiquarks.

The MIT bag energy is given as

$$E_i^{\rm bag} = \sum_q n_q \frac{\Omega_{qi}}{R_i} - \frac{Z_i}{R_i} + \frac{4}{3}\pi R_i^3 B_i(\sigma,\ \sigma^*), \quad (8)$$

where n_q is the number of the constituent quarks (antiquarks) q inside the bag; Z_i is the zero-point motion parameter of the MIT bag and B_i is the bag constant for the hadron i. In the MQMC model, the bag constant is affected by the medium effect, and we adopte the following directly coupling form[16]:

$$B_i(\sigma,\ \sigma^*) = B_0 \exp\left[-\frac{4}{M_i} \left(g_\sigma^{{\rm bag},i} \sigma + g_{\sigma^*}^{{\rm bag},i} \sigma^* \right) \right], \quad (9)$$

with M_i is the vacuum mass of the bag. After the corrections of spurious center of mass motion, the effective mass of a bag is given by[17]

$$M_i^* = \sqrt{{E_i^{\rm bag}}^2 - \langle p_{\rm c.m.}^2 \rangle_i}, \quad (10)$$

with

$$\langle p_{\rm c.m.}^2 \rangle_i = \sum_q n_q^i \left(x_{qi}/R_i \right)^2, \quad (11)$$

in which n_q^i is the number of constituent quark(antiquark) q in hadron i. And the radius R_i of the bag is determined by minimizing the effective mass, which gives

$$\frac{\partial M_i^*}{\partial R_i} = 0. \quad (12)$$

Assume hadronic matter to consist of the members of the SU(3) baryon octet and the kaon doublet. Baryons interact via (σ, ω, ρ, σ^*, ϕ) meson exchanges and antikaons are treated in the same footing. Then the total Lagrangian density of the hadronic matter in the MQMC model can be written as

$$\begin{aligned}
\mathcal{L}_{\rm MQMC} = & \sum_B \bar{\Psi}_B \left[{\rm i}\gamma_\mu \partial^\mu - M_B^* - \left(g_\omega^B \omega_\mu \gamma^\mu \right.\right. \\
& \left.\left. + g_\rho^B \frac{\vec{\tau}_B}{2} \cdot \vec{\rho}_\mu \gamma^\mu + g_\phi^B \phi_\mu \gamma^\mu \right) \right] \Psi_B \\
& + \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \sigma^* \partial^\mu \sigma^* \right) \\
& - \frac{1}{2} \left(m_\sigma^2 \sigma^2 + m_{\sigma^*}^2 \sigma^{*2} - m_\omega^2 \omega_\mu \omega^\mu \right. \\
& \left. - m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - m_\phi^2 \phi_\mu \phi^\mu \right) \\
& - \frac{1}{4} \left(W_{\mu\nu} W^{\mu\nu} + \vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} + F_{\mu\nu} F^{\mu\nu} \right) \\
& + \sum_l \bar{\Psi}_l \left({\rm i}\gamma_\mu \partial^\mu - m_l \right) \Psi_l \\
& + \mathcal{D}_\mu^* K^* \mathcal{D}^\mu K - {M_{\rm K}^*}^2 K^* K, \quad (13)
\end{aligned}$$

where the summation on B is over the octet of baryons (p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-), $l \equiv (e^-,\ \mu^-)$ and the isospin doublet for the antikaons is denoted by $K^* \equiv (K^-,\ \bar{K}^0)$, $W_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\vec{G}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$, $F_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$, $\mathcal{D}_\mu = \partial_\mu + {\rm i} g_\omega^{\rm K} \omega_\mu + {\rm i} g_\rho^{\rm K} \frac{\vec{\tau}_{\rm K}}{2} \cdot \vec{\rho}_\mu + {\rm i} g_\phi^{\rm K} \phi_\mu$. The form of the lagrangian is similar to the usual relativistic mean field Lagrangian[18, 19], except that the effective mass is pre-determined by Eq. (10). The dispersion relation for K^- can be easily derived from the equation of motion, it takes

$$\omega_{{\rm K}^-} = M_{\rm K}^* - \left(g_\omega^{\rm K} \omega_0 + g_\rho^{\rm K} I_{3K} \rho_{03} + g_\phi^{\rm K} \phi_0 \right). \quad (14)$$

For the sake of simplicity, we ignore $\bar{K}^0$ in the present work, and include K^- field only because it is the most

possible one to be s wave condensation ($\vec{k}_{\mathrm{K}}=0$) in dense neutron star matter[18].

From Eqs. (13) and (10), we can derive the equations of motion for the meson fields in uniform static matter:

$$m_\sigma^2\sigma = -\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B} k^2\mathrm{d}k \frac{M_B^*}{\sqrt{k^2+M_B^{*2}}}\frac{\partial M_B^*}{\partial\sigma} - \frac{\partial M_{\mathrm{K}}^*}{\partial\sigma}\rho_{\mathrm{K}}, \tag{15}$$

$$m_{\sigma^*}^2\sigma^* = -\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B} k^2\mathrm{d}k \frac{M_B^*}{\sqrt{k^2+M_B^{*2}}}\frac{\partial M_B^*}{\partial\sigma^*} - \frac{\partial M_{\mathrm{K}}^*}{\partial\sigma^*}\rho_{\mathrm{K}}, \tag{16}$$

$$m_\omega^2\omega_0 = \sum_B g_\omega^B\left(2J_B+1\right)k_B^3/\left(6\pi^2\right) - g_\omega^{\mathrm{K}}\rho_{\mathrm{K}}, \tag{17}$$

$$m_\rho^2\rho_{03} = \sum_B g_\rho^B I_{B3}\left(2J_B+1\right)k_B^3/\left(6\pi^2\right) - g_\rho^{\mathrm{K}}\rho_{\mathrm{K}}, \tag{18}$$

$$m_\phi^2\phi_0 = \sum_B g_\phi^B\left(2J_B+1\right)k_B^3/\left(6\pi^2\right) - g_\phi^{\mathrm{K}}\rho_{\mathrm{K}}, \tag{19}$$

where J_B and k_B are the spin projection and the fermi momentum for baryon B, respectively. Using

$$\frac{\partial M_i^*}{\partial\sigma} = \left.\frac{\partial M_i^*}{\partial\sigma}\right|_{R_i} + \left.\frac{\partial M_i^*}{\partial R_i}\right|_{\sigma}\frac{\partial R_i}{\partial\sigma}$$

and Eq. (12), we can give the differentiation of the effective hadron (baryon and kaon) species mass with scalar field σ:

$$\frac{\partial M_i^*}{\partial\sigma} = \frac{E_i^{\mathrm{bag}}\dfrac{\partial E_{\mathrm{bag}}^i}{\partial\sigma} - \dfrac{1}{2}\dfrac{\partial\langle p_{\mathrm{c.m.}}^2\rangle_i}{\partial\sigma}}{M_i^*}, \tag{20}$$

$$\frac{\partial E_i^{\mathrm{bag}}}{\partial\sigma} = \sum_q \frac{n_q}{R_i}\frac{\partial\Omega_{qi}}{\partial\sigma} + \frac{4}{3}\pi R_i^3\frac{\partial B_i}{\partial\sigma}, \tag{21}$$

$$\frac{\partial\langle p_{\mathrm{c.m.}}^2\rangle_i}{\partial\sigma} = \frac{2}{R_i^2}\sum_q n_q\left(\Omega_{qi}\frac{\partial\Omega_{qi}}{\partial\sigma} + R_i^2 g_\sigma^q m_q^*\right), \tag{22}$$

$$\frac{\partial\Omega_{qi}}{\partial\sigma} = -R_i g_\sigma^q\frac{\Omega_{qi}/2 + m_q^* R_i\left(\Omega_{qi}-1\right)}{\Omega_{qi}\left(\Omega_{qi}-1\right)+m_q^* R_i/2}, \tag{23}$$

and the differentiation with respect to σ^* is likewise.

Since the time scale of a star can be regarded as infinite compared to the typical time for weak interaction, which violates the strangeness conservation, the strangeness quantum number is therefore not conserved. While the β equilibrium should be maintained. All the β equilibrium conditions involving the baryon octet

$$\begin{array}{lll} p+e^-\leftrightarrow n+\nu_e, & \Lambda\leftrightarrow n, & \\ \Sigma^+ + e^-\leftrightarrow n+\nu_e, & \Sigma^0\leftrightarrow n, & \Sigma^-\leftrightarrow n+e^-+\bar{\nu}_e, \\ & \Xi^0\leftrightarrow n, & \Xi^-\leftrightarrow n+e^-+\bar{\nu}_e \end{array}$$

may be summarized by a single generic equation:

$$\mu_B = \mu_n - q_B\mu_e, \tag{24}$$

where μ_B and q_B are, respectively, the chemical potential and electric charge of baryon B with

$$\mu_B = \sqrt{k_B^2+M_B^{*2}} + g_\omega^B\omega_0 + g_\phi^B\phi_0 + g_\rho^B I_{3B}\rho_{03}. \tag{25}$$

From the decay modes

$$K^-\leftrightarrow e^-+\bar{\nu}_e, \qquad \mu^-\leftrightarrow e^-+\bar{\nu}_e+\nu_\mu,$$

we know that when the effective energy of K^- meson, ω_{K^-}, equals to its chemical potential, μ_{K^-}, which in turn is equal to the electrochemical potential μ_e, K^- condensation is formed, i.e.

$$\omega_{\mathrm{K}^-} = \mu_e = \sqrt{k_e^2+m_e^2} = \mu_\mu = \sqrt{k_\mu^2+m_\mu^2}. \tag{26}$$

Note that the first equal sign in Eq. (26) is only valid when the condensation takes place. And there are two physical constraints on the HP phase left, they are the conservation of baryon-number and electric charge, which are

$$\rho_{\mathrm{HP}} = \frac{1}{6\pi^2}\sum_B b_B\left(2J_B+1\right)k_B^3, \tag{27}$$

$$\rho_{\mathrm{HP}}^{\mathrm{ch}} = \frac{1}{6\pi^2}\sum_B q_B\left(2J_B+1\right)k_B^3 + \frac{1}{3\pi^2}\sum_l q_l k_l^3 - \rho_{\mathrm{K}}. \tag{28}$$

The electric charge neutrality condition for the pure HP phase is

$$\rho_{\mathrm{HP}}^{\mathrm{ch}} = 0. \tag{29}$$

Therefore, the energy density and pressure for the HP are:

$$\begin{aligned}\mathcal{E}_{\rm HP} &= \frac{1}{2}\left(m_\sigma^2\sigma^2 + m_{\sigma^*}^2\sigma^{*2} + m_\omega^2\omega_0^2 + m_\rho^2\rho_{03}^2 + m_\phi^2\phi_0^2\right) \\ &+m_{\rm K}^*\rho_{\rm K} + \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B}\sqrt{k^2+M_B^{*2}}k^2{\rm d}k \\ &+\frac{1}{\pi^2}\sum_l \int_0^{k_l}\sqrt{k^2+m_l^2}k^2{\rm d}k, \end{aligned} \tag{30}$$

$$\begin{aligned}\mathcal{P}_{\rm HP} &= \frac{1}{2}\left(m_\omega^2\omega_0^2 + m_\rho^2\rho_{03}^2 + m_\phi^2\phi_0^2 - m_\sigma^2\sigma^2 - m_{\sigma^*}^2\sigma^{*2}\right) \\ &+\sum_B \frac{2J_B+1}{6\pi^2}\int_0^{k_B}\frac{k^4{\rm d}k}{\left(k^2+M_b^{*2}\right)^{1/2}} \\ &+\frac{1}{3\pi^2}\sum_l \int_0^{k_l}\frac{k^4{\rm d}k}{\left(k^2+m_l^2\right)^{1/2}}. \end{aligned} \tag{31}$$

We assume the quark matter phase may occur in the core of star in the form of unpair quark matter (UQM) and the spontaneous broken chiral symmetry is restored so the quarks take their current masses. To describe the UQM, the MIT bag model is adopted, where the quarks are confined in a giant bag without dynamic freedom. If the bag constant for UQM is B, the energy, pressure, baryon number and electric charge densities at zero temperature are given by

$$\begin{aligned}\mathcal{E}_{\rm UQM} &= \frac{3}{\pi^2}\sum_q \int_0^{k_q}\sqrt{k^2+m_q^2}k^2{\rm d}k \\ &+\frac{1}{\pi^2}\sum_l \int_0^{k_l}\sqrt{k^2+m_l^2}k^2{\rm d}k + B, \end{aligned} \tag{32}$$

$$\begin{aligned}\mathcal{P}_{\rm UQM} &= \frac{1}{\pi^2}\sum_q \int_0^{k_q}\frac{k^4{\rm d}k}{\left(k^2+m_q^2\right)^{1/2}} \\ &+\frac{1}{3\pi^2}\sum_l \int_0^{k_l}\frac{k^4{\rm d}k}{\left(k^2+m_l^2\right)^{1/2}} - B, \end{aligned} \tag{33}$$

$$\rho_{\rm UQM} = \frac{1}{3\pi^2}\sum_q k_q^3, \tag{34}$$

$$\rho_{\rm UQM}^{\rm ch} = \frac{1}{\pi^2}\sum_q q_q k_q^3 + \frac{1}{3\pi^2}\sum_l q_l k_l^3 \tag{35}$$

with q_q is the electric charge for quark q. The exact value of B is not fixed till now, and the phase transition point from HP to UQM depends on its value sensitively, which will be discussed later.

Chemical equilibrium among the quark flavors and the leptons is maintained by the following weak reactions:

$$d \leftrightarrow u + e^- + \bar{\nu}_e, \quad s \leftrightarrow u + e^- + \bar{\nu}_e, \quad s + u \leftrightarrow d + u.$$

we can get the equilibrium condition for the pure quark matter phase

$$\mu_d = \mu_s = \mu_u + \mu_e, \tag{36}$$

where

$$\mu_q = \sqrt{m_q^2 + k_q^2} \tag{37}$$

is the chemical potential for the quark q, and can be obtained by the β equilibrium in mixed state. For the state where HP and UQM coexist, i.e. the mixed phase, the quark chemical potentials for a system in chemical equilibrium are related to those for baryon and electron by[20]

$$\mu_u = \frac{1}{3}\mu_n - \frac{2}{3}\mu_e, \tag{38}$$

$$\mu_d = \mu_s = \frac{1}{3}\mu_n + \frac{1}{3}\mu_e. \tag{39}$$

Global electric charge neutrality condition must be satisfied and the Gibbs construction requires that the pressures of two phases should be equal at zero temperature. If the volume fraction of UQM phase is χ, then coexisting conditions are

$$\chi\rho_{\rm UQM}^{\rm ch} + (1-\chi)\,\rho_{\rm HP}^{\rm ch} = 0, \tag{40}$$

$$\mathcal{P}_{\rm UQM} = \mathcal{P}_{\rm HP}. \tag{41}$$

The energy density and the total baryon-number density read

$$\mathcal{E} = \chi\mathcal{E}_{\rm UQM} + (1-\chi)\,\mathcal{E}_{\rm HP}, \tag{42}$$

$$\rho = \chi\rho_{\rm UQM} + (1-\chi)\,\rho_{\rm HP}. \tag{43}$$

We take $m_u = m_d = 0$, $m_s = 130$MeV[21]. The bag constants and zero-point motion parameters are calibrated to reproduce the mass spectrum and the stable condition Eq. (12) for the MIT-bags in free space. Assuming the nucleon's radius to be 0.6fm,

Table 1: The zero-point motion parameters Z_i and radii R_i are obtained to reproduced the mass spectrum in vacuum and Eq. (12). And that $B_0^{1/4}=$ 188.2385MeV has been fixed by the properties of nucleon. The mass spectrum adopted here is taken from Ref[21].

	M(MeV)	Z	R(fm)
N	939.0	2.0314	0.6000
Λ	1115.7	1.7913	0.6472
Σ^+	1189.4	1.6124	0.6731
Σ^0	1192.6	1.6041	0.6742
Σ^-	1197.4	1.5919	0.6758
Ξ^0	1314.8	1.4439	0.6940
Ξ^-	1321.3	1.4262	0.6960
K^-	493.7	1.1632	0.3391

the bag constant B_0 in vacuum for the nucleon can be fitted together with the mass 939MeV. The result is $B_0^{1/4}=188.2385$MeV. In Table 1, the zero-point motion parameters and bag-radii for baryons and K^- are listed. And the mass spectrum for mesons transferring interactions are listed in Table 2.

The σ, ω and ρ mesons couple only to the up and down quarks while σ^* and ϕ couple to the strange quark. We thus set

$$g_\sigma^s = g_\omega^s = g_\rho^s = g_{\sigma^*}^u = g_{\sigma^*}^d = g_\phi^u = g_\phi^d = 0.$$

By assuming the SU(6) symmetry of the simple quark model we have the relations[14]

$$\begin{array}{ll}
g_\sigma^u = g_\sigma^d \equiv g_\sigma^{u,d}, & g_{\sigma^*}^s = \sqrt{2} g_\sigma^{u,d}, \\
g_\sigma^i = \left(n_u^i + n_d^i\right) g_\sigma^{u,d}, & g_{\sigma^*}^i = \sqrt{2} n_s^i g_\sigma^{u,d}; \\
g_\omega^u = g_\omega^d \equiv g_\omega^{u,d}, & g_\phi^s = \sqrt{2} g_\omega^{u,d}, \\
g_\omega^i = \left(n_u^i + n_d^i\right) g_\omega^{u,d}, & g_\phi^i = \sqrt{2} n_s^i g_\omega^{u,d}; \\
g_\rho^u = g_\rho^d \equiv g_\rho^{u,d}, & g_\rho^i = g_\rho^{u,d}; \\
g_\sigma^{\mathrm{bag},i} = \dfrac{1}{3}\left(n_u^i + n_d^i\right) g_\sigma^{\mathrm{bag},N}, & g_{\sigma^*}^{\mathrm{bag},i} = \dfrac{\sqrt{2}}{3} n_s^i g_\sigma^{\mathrm{bag},N}.
\end{array}$$

Then there are only four independent constants of coupling left. Three of them are the couplings between light quarks and nonstrange meson mean fields, i.e. $g_\sigma^{u,d}$, $g_\omega^{u,d}$ and $g_\rho^{u,d}$. The last one is $g_\sigma^{\mathrm{bag},N}$ measuring the interaction between the bag constant and the scalar σ mean fields. We adjust them to reproduce the saturation properties of nuclear matter: the symmetric energy index a_{sym}=32.5MeV, the binding energy $E_b=-16$MeV and the compressibility K=289MeV at the density ρ_0=0.17fm^{-3}. The four independent coupling constants are listed in Table 3.

Table 2: The mass spectrum (in MeV) for mesons transferring interactions[21].

m_σ	m_ω	m_ρ	m_{σ^*}	m_ϕ
550	783	776	980	1020

Table 3: The four independent coupling constants

$g_\sigma^{u,d}$	$g_\omega^{u,d}$	$g_\rho^{u,d}$	$g_\sigma^{\mathrm{bag},N}$
0.9668	2.6992	7.9327	6.8369

The hadron, lepton and quark population at different baryon-densities in neutron star matter with and without UQM respectively, are shown in Figure 1. The bag constant for UQM is fixed at $B^{1/4}$=180MeV. Figure 1(a) tells us that when the density reaches $1.6\rho_0$ mixed phase appears. The critical density obtained here for phase transition from pure hadronic matter to mixed phase is similar to those reported by other models, such as that by FST model in Ref.[6] or the result by QMC model in Ref.[11]. While in the present model hyperons seem to appear more easily than that in QMC model. The reason is that the effective masses of hyperons in MQMC model are lower than that in QMC model because in the MQMC model the bag constants of hadrons keep decreasing as the density increases. When ρ_B=7.8ρ_0, the volume of hadronic matter go down to zero and

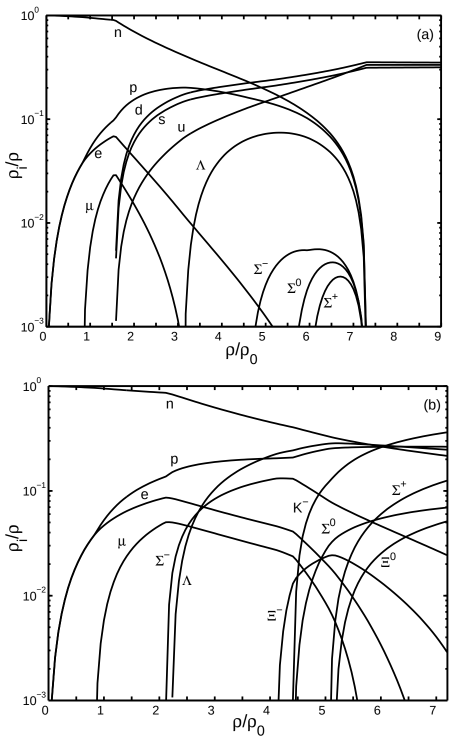

Figure 1: The hadron, lepton and quark population at different baryon-densities in a system compose of (a) HP+UQM with $B^{1/4}$ =180MeV, (b) pure HP

the pure UQM exists.

In the present work, the negative charged kaon is also taken into account, however we cannot find K^- in Figure 1(a). To illustrate the fact, let's look at Figure 1(b), which shows the population of compositions in pure HP. It can be found that K^- begins to condense at a critical density of about $4.4\rho_0$ which is larger than the critical density to mixed phase. Therefore we can learn that K^- condensation is suppressed because of the deconfinement mechanism. The fact is that the presence of UQM lowers electrochemical potential than that in pure HP and therefore will force the critical point of condensation to a higher density. And it is clear that the critical point has already been forced into the region where pure UQM exists without any hadrons. Our calculations indicate that for any choice of the

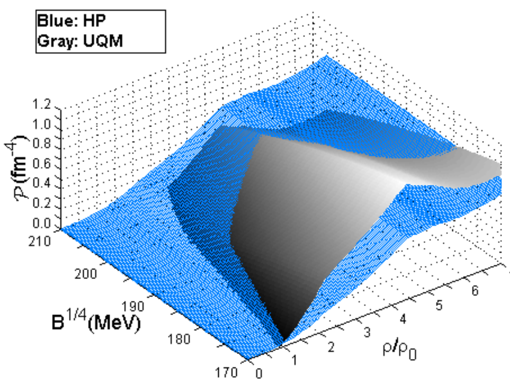

Figure 2: Pressure as a function of nuclear density and bag constant. The light blue one is for pure HP and that gray one is for pure UQM using relations (38) and (39).

bag constant the condensed phase is suppressed once the deconfinement phase transition is attainable, i.e. when $B^{1/4}$ <202.2MeV which we know from Figure 2. Our result is different from that in Ref.[6], where the kaon condensation appears within the mixed phase at $9.26\rho_0$ for $B^{1/4}$=185MeV. But the condensed point is so high that the authors have to concluded that K^- condensation would not come along with the neutron star also.

The relation between the bag constant B and the critical point of deconfinement is shown in Figure 2. The light blue surface represents the pressure as a function of ρ and B for HP, and the gray one is for UQM with the conditions (38) and (39). The figure reveals that when $B^{1/4}$ <202.2MeV, the two surfaces can always have intersection as the density increases, which means the system would enter the mixed phase at the matching point. When $B^{1/4}$ is greater than about 202.2MeV, no intersection appears at all possible densities in the interior of neutron star which means no hadron will deconfines and the behaviors of compositions are shown in Figure 1(b). For a given B, pressure of quark matter phase increases firstly till it reaches a maximum point then it drops as the density increases. We found that the maximum is at the critical density for K^- condensation and therefore with any value of $B^{1/4}$ <202.2MeV deconfiment

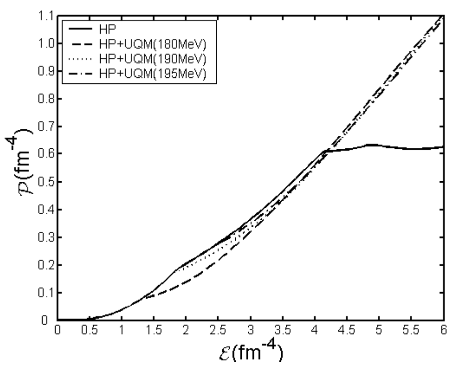

Figure 3: EOSes for HP and HP+UQM with different bag constants B.

position is always lower than that for K^- condensation. Moreover, from the figure we find that the critical density for deconfinement is sensitive to the value of B.

The EOSes for pure HP and HP+UQM are shown in Figure 3. For pure HP, the first turning point corresponds to the appearance of hyperons. And when the density reaches $4.4\rho_0$, K^- begins to condense, consequently the EOS is softened significantly. In the system of HP+UQM, hyperons are forced to appear at higher densities. At the low energy density, the EOS for HP+UQM is softer than that of pure HP because of the deconfinement phase transition. However, after the K^- condensation takes place in the pure HP, the case is contrary, which can be interpreted by two facts: First, the abundance of hyperon is higher for pure HP at the same energy density; Second, the s wave K^- condensation contributes only to the total energy but not to pressure because of the zero momentum at ground state.

The radius-mass relationships of static neutron star obtained by solving the Tolman-Oppenheimer-Volkoff equations[22] are shown in Figure 4(a) for different equations of state. For all the cases studied here, the maximum masses of the stars are found to lie between $1.45M_\odot$ and $1.52M_\odot$ which are all larger than the best measured pulsar mass $1.44M_\odot$ in the binary pulsar PSR 1913+16[24]. Furthermore, for the case of HP+UQM, a smaller bag constant gives a lower maximum mass, so the EOSes with about

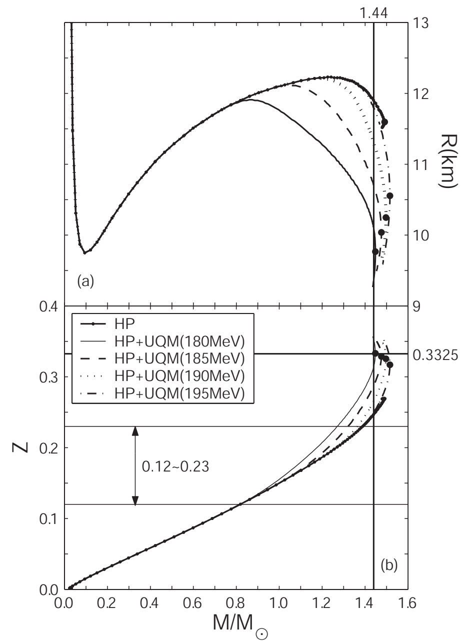

Figure 4: (a)The radii versus masses for neutron stars for different EOSes. The dots show the positions for maximum masses.The vertical line shows the maximum mass limit by PSR 1913+16[24]. (b)The gravitational redshift versus masses for neutron stars. The two lower horizontal lines denote the observational value for the gravitational redshift of the neutron star 1E 1207.4-5209[23] and the one lies on 0.3325 is the lower limit for maximum redshift[25, 26].

$B^{1/4} < 180$MeV should be ruled out.

The gravitational redshifts are plotted in Figure 4(b). A redshift of z=0.35[25], with a total measurement error of order of 5%[26], was inferred by identifying three sets of redshifted transitions in the EXO0748-676 spectrum, so it imposes a lower limit of about 0.3325 to the maximum redshift. From the figure, we see that the EOS of pure hadron matter with condensed K^- phase is ruled out, and EOSes of HP+UQM with $B^{1/4}$ more than 180MeV would be ruled out likewise, but that with 180MeV is marginally permitted because it produces a maximum redshift of 0.3330. So the value of $B^{1/4}$ is constrained to be about 180MeV by the combined con-

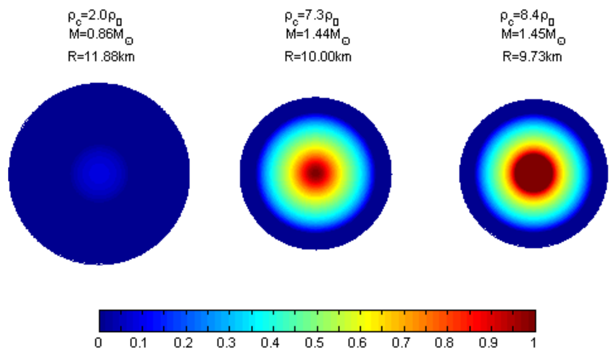

Figure 5: Phase structure of hybrid stars with bag constant fixed at $B^{1/4}$=180MeV. The color shows the volume fraction χ of quark matter, and that for hadronic matter is $1-\chi$. Above the star, its properties are marked.

straint from PSR 1913+16 and EXO0748-676. Sanwal et al. discussed the absorption lines from the neutron star 1E 1207.4-5209, where a redshift of 0.12∼0.23 was yielded if the observed features are identified as atomic transitions of once-ionized helium in a strong magnetic field[23]. By the EOS of HP+UQM with $B^{1/4}$ = 180MeV, we can see that the identification corresponds to that mass M= 0.82 ∼1.27$M_\odot$ and radius $R = 11.0 \sim 11.9$km respectively. These values appear to be realistic.

In Figure 5, the phase structure in possible hybrid stars are figured out, where $B^{1/4}$=180MeV is fixed. When $\rho_c < 8.4\rho_0$, the neutron star is stable since $\frac{dM}{d\mathcal{E}_c} > 0$, where $\mathcal{E}_c$ is the energy density at the center. In the possible hybrid star the density increases as going deeply into the interior and the mixed phase exists within some critical radius. Within the hadronic matter crust the phase transition from normal nuclear matter into hyper-nuclear matter may occur, but K^- condensation phase is suppressed. For different hybrid stars, the volume fraction for UQM keeps increasing, whereas the pure hadronic matter crust becomes thinner and thinner as the central density rises. Especially when ρ_c goes up to about $7.3\rho_0$, a core of pure quark matter comes into being. And the quark core will expands further as the ρ_c increases. For the neutron star with maximum mass, an evident quark matter core is presented, which is described by the third pattern. The appearance of the pure quark core is notable. Many other works have been carried out to study the quark matter phase within the three-flavor NJL model, but all of them are unable to construct a stable hybrid star with pure quark core[27] and only a star with mixed phase core is possible[28].

Recently, Ransom, et. al. inferred that at least one of the stars in Terzan 5 is more massive than 1.48, 1.68, or 1.74 $M_\odot$ at 99%, 95%, and 90% confidence levels[29]. While compared with the limit of 1.68$M_\odot$, all the EOSes with exotic phases presented here would be ruled out. Therefore, if the rather massive star is confirmed condensed K^- phase and deconfiment phase on unpaired state are likely to be denied in neutron stars.

In summary, we have investigated the K^- condensation and the deconfinement phase transition in the frame of MQMC model. The model predicts a critical density for kaon condensation in pure HP. When UQM exists, which is only possible for $B^{1/4}$ <202.2MeV, condensed phase is suppressed. We find that only the EOS of HP+UQM with $B^{1/4}$ about 180MeV can fit the observational mass of star PSR 1913+16 and the inferred redshift for EXO0748-676 at the same time. The phase structures of possible hybrid stars with different central densities are discussed, it is found that for EOS of HP+UQM with $B^{1/4}$ =180MeV a star with central density higher than $7.3\rho_0$ will has a pure quark core and the pure hadronic matter star exists when $\rho_c < 1.6\rho_0$. Between the two densities, the star is characterized by a crust of hadronic matter and a core of mixed phase.

The recent inferred mass of the star Terzan 5 I is also considered, and it is found that the mass limit of 1.68$M_\odot$ at 95% confidence level makes all the EOSes presented here ruled out. Therefore, if this rather massive star is confirmed, condensed K^- phase and deconfined phase in unpaired state are unlikely to appear in neutron star by the present model and accordingly the matter of hadrons in normal state seems to be preferred as claimed by Özel[30]. Does this really mean that the ground state of matter is composed of normal nuclear matter without exotic phases? Actually, we should note that in the present

model all the octet of baryons are included in the HP but quarks may be deconfined within the matter of nucleons without hyperons. Furthermore, the deconfined quarks are in the unpaired state in the present calculation where the quark-quark interactions are neglected, but quarks could be in the color superconducting state as well if the the attractive interaction in color antitriplet channel is considered. Therefore the possibility of constructing an EOS with exotic phases which satisfies the observational constraints could not be eliminated, which deserves further investigations.

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